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ULTIMATE STRENGTH ANALYSIS OF STRUCTURAL CONCRETE DEEP BEAMS USING STRUT-TIE MODELS

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ABSTRACT

This paper presents an ultimate strength analysis of two reinforced and one prestressed concrete deep beams tested to failure. A nonlinear strut-tie model approach implemented with an interactive computer graphics program was utilized to evaluate the ultimate strength and nonlinear behavior of the beams. Different types of strut-tie models for the beams were selected based on the principal compressive stress trajectories, actual specimen detailing, and loading and support conditions. The present study shows that the nonlinear strut-tie model approach can provide simple and effective solutions for a large number of analysis situations by describing the essential structural behavior aspects and evaluating the strength of structural concrete. It also allows for the conceptual representation of the complex concrete and reinforcing steel interactions, and permits the study of localized effects through the bearing capacity evaluation of nodal zones. The framework provided by the nonlinear strut-tie model approach for considering combined actions is strongly suggested in the ultimate strength analysis of structural concrete deep beams.

Keywords: deep beams, nonlinear strut-tie model approach, computer graphics, strength and nonlinear behavior, bearing capacity evaluation

1. INTRODUCTION

Strut-tie models have proved to be useful for the design and detailing of disturbed regions (so called “D” regions) of reinforced and prestressed concrete structures. These models represent the load-carrying mechanism of a concrete member by approximating the internal force flow by means of struts representing the flow of principal compressive stresses and ties representing the principal tensile reinforcement. Strut-tie models are a generalization of the truss analogy, which originally appeared in the early 1900’s, and became the basis of the current 45-degree truss model used for the design of beam-type regions (so called “B” regions). The truss analogy concept [14,18], which assumes concrete after cracking is not capable of resisting tension, postulates a cracked reinforced concrete beam acts as a truss with parallel longitudinal chords and a web composed of diagonal concrete struts and transverse ties. This method was later refined and expanded in the 1960's [8,11], and the

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scientific basis for its rational application in tracing the concept back to the theory of plasticity was created in the 1970's and early 1980's [10,12,13,15,16,22]. By considering deformations and realistic stress-strain relationships for concrete and reinforcement, truss models have been refined to predict the response of cracked reinforced concrete members subjected to shear, torsion, and combined actions [5,23]. In the late 1980's, the truss model for beam-type regions was extended to all parts of the structure in the form of strut-tie systems, Ref. [19]. Since then, extensive applications of strut-tie models to analyze and design structural concrete have been developed [1,2,6,17,20,21,24]. Recently, an interactive computer graphics program NL-STM that implements strut-tie models has been developed [26].

In this paper, the strength and nonlinear behavior of two reinforced and one prestressed concrete deep beams tested to failure were evaluated using a nonlinear strut-tie model approach [27]. In this approach, nonlinear techniques are incorporated in the selection, analysis, and verification processes of the strut-tie models. The additional positioning of concrete ties and steel struts at the locations of steel ties and concrete struts, respectively, is also incorporated in the approach. An interactive computer graphics program NL-STM implementing the nonlinear strut-tie model approach was used for the analysis. Strut-tie models reflecting the actual support and loading conditions were selected for the beams. Different values for the effective strengths of the concrete struts were determined and the bearing capacities of critical nodal zones were verified using failure criteria incorporating the different stress states.

Modeling reinforced and prestressed concrete members using finite element nonlinear analysis techniques alone for design or analysis purposes is difficult due to the complex interaction between concrete and reinforcing steel, including cracking phenomena. In addition, even if such an analysis is carried out, the problem of transposing the analysis results into an arrangement of steel and concrete remains. The nonlinear strut-tie model approach provides an innovative bridge between current sophisticated analysis techniques and actual structural dimensioning. The approach facilitates the conceptual representation of complex concrete and reinforcing steel interactions, while also allowing the study of localized effects through the bearing capacity evaluation of nodal zones. This paper reviews the performance of a nonlinear strut-tie model approach aided with a graphics program in the evaluation of the strength and nonlinear behavior of structural concrete deep beams as well as in the verification of the bearing capacities of critical nodal zones.

2. NONLINEAR STRUT-TIE MODEL APPROACH

Selecting a strut-tie model using the conventional strut-tie model approach is an iterative process. The process starts by selecting the initial centerline truss model, followed by an evaluation of the member forces. Components of the model are then dimensioned based on the internal member forces and strengths of the concrete and steel, and forming and analyzing the nodal zones. If the cross-sectional areas of two almost parallel concrete struts placed side by side overlap each other or the dimensioned strut-tie model is not compatible with the actual size of the structural concrete, the truss model itself and/or its geometry must be modified and the procedure repeated until a satisfactory solution is obtained.

Selecting a strut-tie model using the nonlinear strut-tie model approach is also an iterative process. However, unlike the conventional strut-tie model approach, the nonlinear strut-tie model approach incorporates nonlinear techniques in the selection, analysis, and verification processes of a strut-tie model to eliminate the limitations of the conventional strut-tie model approach relating to behavior and strength predictions of structural concrete and the design of structural concrete which experiences nonlinear behavior. The approach also incorporates additional positioning of concrete ties and steel struts at the locations of steel ties and concrete struts, respectively. Detail descriptions on the approach were provided in Ref. [27]. A flow chart for the nonlinear strut-tie model approach is shown in Figure 1.

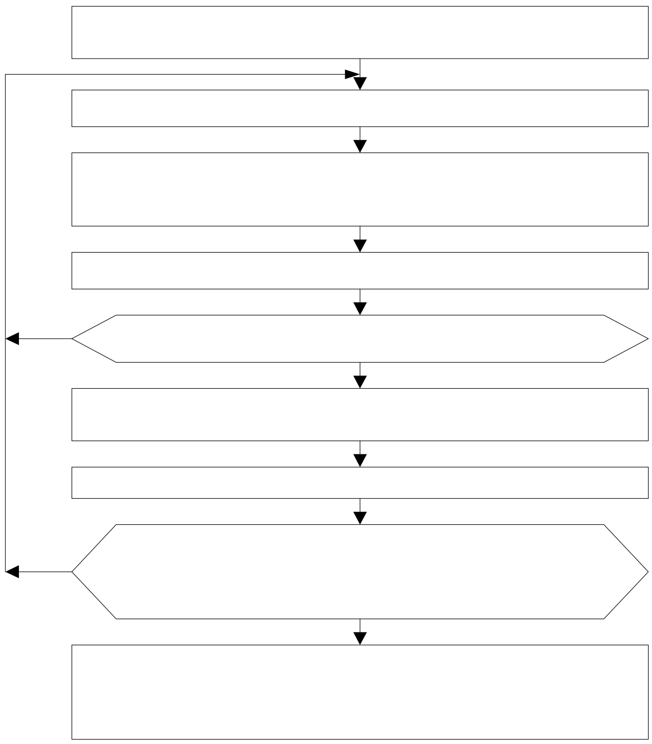


Figure 1. Flow chart for the nonlinear strut-tie model approach

3. SUMMARY OF PREVIOUS SHEAR TEST RESULTS

3.1 Reinforced Concrete Beams

Two rectangular beams with different stirrup detailing, fabricated and tested to failure at Purdue University [4], are considered in this study. The shear span to depth ratio of the beams was equal to 2.15. The tensile reinforcement consisted of 2D29 and 2D25 deformed bars arranged in two layers. The compression reinforcement consisted of 2D25 deformed bars. The detailing for the specimens and the strain gage locations on the stirrup legs and longitudinal bars are shown in Figure 2. Detailed specimen information is given in Table 1(a).

Table 1: Material information

(a): reinforced concrete beams

Specimen		**Beam 1**		**Beam 10**	
f_c' (MPa)		**38.27**		**30.36**	
Bars & Stirrup		2D25	2D29	2D25	2D29
Top Steel	E_o	199,400	-	189,300	-
	f_y	419.2	-	507.1	-
	ε_y	0.0021	-	0.0032	-
Bottom Steel	E_o	199,400	185,900	189,300	185,900
	f_y	419.2	490.2	507.1	493.6
	ε_y	0.0021	0.0029	0.0032	0.0030
Web Steel (D10)	E_o	202,800		213,000	
	f_y	524.0		534.1	
	ε_y	0.0045		0.0045	

Unit for E_o and f_y is MPa.

(b) pre-tensioned concrete beams

Concrete	-	Transfer	Test
	f_c' (MPa)	40.27	60.74
	E_c (MPa)	38,750	39,510
	f_r' (MPa)	6.34	-
Prestressing Strand (Gr. 270)	-	Top	Bottom
	A_{ps} (cm^2)	1.05	1.05
	d_p',d_p (cm)	5.10	66.00
	E_{ps} (MPa)	192,500	192,500
	f_{pu} (MPa)	1,940	1,940
	f_{si} (MPa)	1,430	1,430
	F_{se} (MPa)	1,380	1,290
	P_e (kN)	290.5	1091.6
Mild Reinf.	-	D16 Bar	D13 Bar
	A_s' , A_v (cm^2)	2.00	1.23
	E_s (MPa)	200,100	203,400
	f_y (MPa)	441.3	358.5

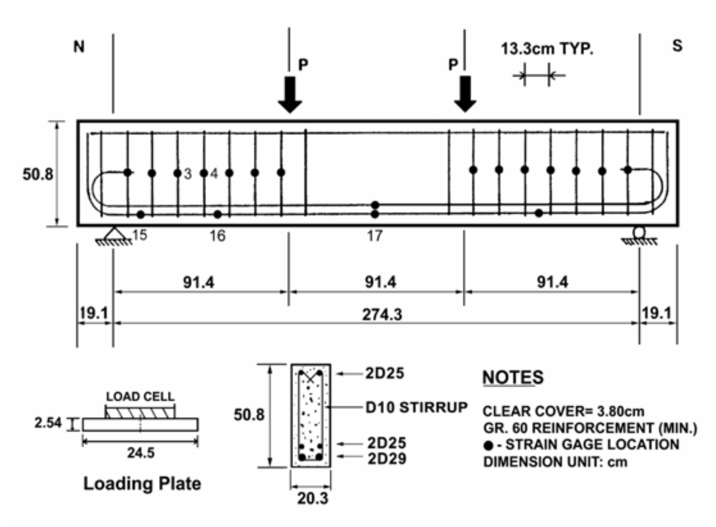
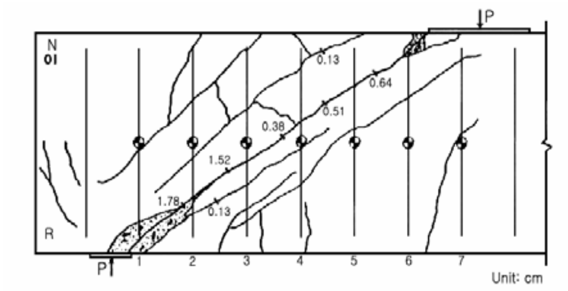
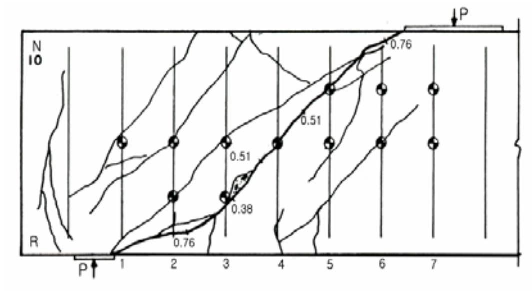


Figure 2. Test specimens of reinforced concrete beams (Adapted from Ref. [4])



(a) Beam 1



(b) Beam 10

Figure 3. Detailed crack plots for reinforced concrete beams (Adapted from Ref. [4])

In Beam 1, flexural cracks appeared first in the constant moment region at 45kN. At 178kN both flexural-shear cracks formed and web-shear cracks developed at 222kN. Final failure occurred at a load of 479kN when the web-shear crack extended from the support to the point load. Figure. 3(a) shows the detailed crack pattern at failure for the north side of the beam. The targets indicate the location of the strain gages in the stirrup reinforcement. The shaded region near the applied load represents the concrete failure zone.

In Beam 10, U-stirrups with standard 135-degree hooks developed around the compression steel were used. The failure crack pattern on the north side of the beam is shown in Figure. 3(b). The first flexural cracking formed at 89kN. At 133kN, some of the flexural cracks in the shear span turned into inclined cracks. Other web-shear cracks developed at 178kN. An increased load led to shear crack growth toward the support and point load, and final failure occurred at the north shear span. The failure load was 387kN. The yielding of the stirrup reinforcement was only observed in the vicinity of the lower part of the diagonal shear crack at gage locations 2, 3, and 4.

3.2 Prestressed Concrete Beam

The test specimen, Beam I-4A, was a full scale, pretensioned AASHTO Type I beam with a span to depth ratio of 4.28, Ref. [7]. The nominal beam dimensions and strain gage locations are shown in Figure 4. Strain gages were attached to the prestressing strand and mild reinforcement. Before placing the strain gages, the strands were tensioned to 22kN. Following instrumentation, the strands were stressed to 151kN ($0.75 f_{pu} A_{pu}$). Detailed specimen information is given in Table 1(b).

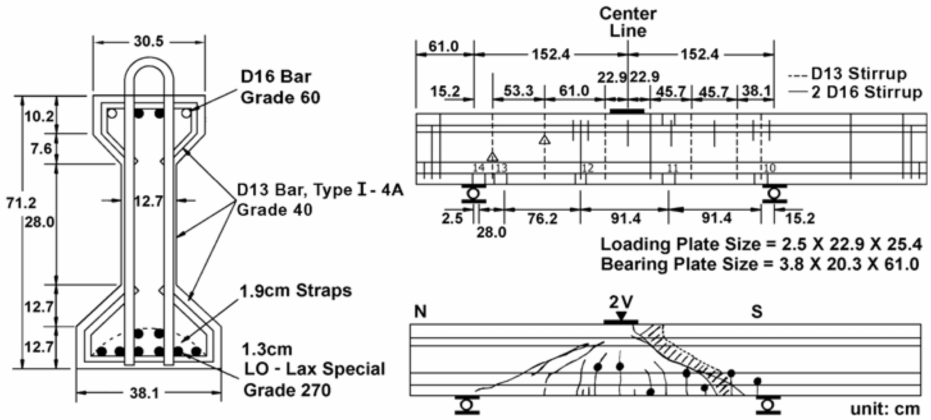


Figure 4. Test specimen of pre-tensioned concrete beam (Adapted from Ref. [7])

In the test, the first diagonal crack opened in the S-shear span at a shear of 525kN. This was followed by a diagonal crack in the N-shear span at a shear of 534kN. The longitudinal strands showed no signs of slip up to failure. Failure started with an initial spalling of the concrete under the edge of the loaded plate in the S-shear span, followed by web crushing at a shear force of 718kN. The failure zone is identified by the shaded region in Figure 4. The yielding of the stirrup reinforcement was observed upon the formation of the inclined shear crack. The strain measurements for the strands indicated no bond deterioration as the shear force approached failure level.

4. ANALYTICAL EVALUATION OF TEST RESULTS

A strut-tie model analysis of the three test specimens was conducted following the procedure suggested by the nonlinear strut-tie model approach. The development of the strut-tie models and an evaluation of the test results of the three deep beams were carried out. The bearing capacities of critical nodal zones were verified using a finite element nonlinear analysis, which included failure criteria [9] incorporating the different biaxial stress states. The nodal zones were considered to be two-dimensional stress fields whose boundaries were determined by the intersection of the stress fields framing into the nodes. More than 20 incremental loading steps were used in the analysis of critical nodal zones.

4.1 Beam 1

1) Selection of Strut-Tie Model

In selecting the strut-tie model for this beam, the location of the steel reinforcement was used to determine the locations of steel ties, while the principal compressive stress trajectories were used to determine the locations and orientations of the struts. A finite element analysis of the plain concrete beam indicated each point load was carried to the nearest support by a single diagonal strut and arch action due to the stirrup reinforcement, as shown in Figure 5.

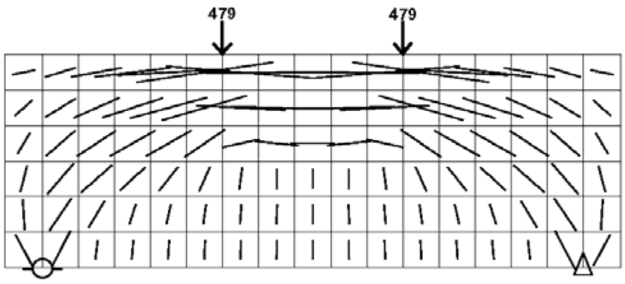


Figure 5. Principal compressive stress flows of Beam 1

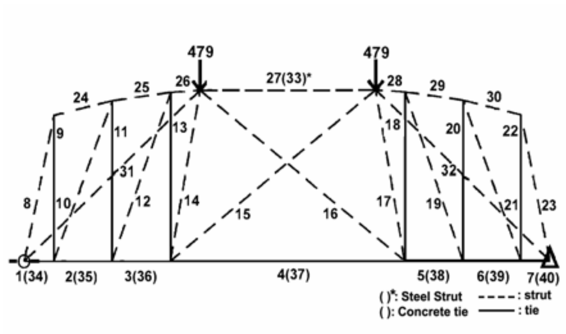


Figure 6. Strut-tie model for Beam 1

Figure 6 shows the strut-tie models selected for this beam. The arch was formulated by multiple concrete struts. The flexural compression zone between the two point loads was modeled by a concrete strut and steel strut with a cross-sectional area of $10.2cm^2$. The shear spans were divided into three equal zones. The transverse steel ties with cross-sectional areas of $3.3cm^2$ for members 9, 11, 20, and 22, and $3.4cm^2$ for members 13 and 18 were located at the center of each zone representing the results of the stirrups in that zone. The longitudinal steel ties with cross-sectional areas of $23.1cm^2$ were placed at the centroid of the longitudinal reinforcements. Cross members 15 and 16 were placed to stabilize the model. In the strut-tie model shown in Figure 6, additional longitudinal concrete ties with cross-sectional areas of 314.8cm^2 were placed at the locations of the longitudinal steel ties. The cross-sectional areas of the concrete ties were determined by multiplying the width of the beam by the effective widths of the concrete ties [27].

2) Material Strength and Dimensioning of Strut-Tie Model

The cross-sectional areas of the concrete struts were determined using the algorithm [27] that only required a few iterations within the effective strength limits. The effective strength levels of the concrete struts were determined using the procedure proposed in Ref. [25], wherein the principal stress ratios of the finite elements modeling the struts were implemented and the degree of confinement in relation to the reinforcement details was considered. Table 2 lists the effective strength of the concrete struts in the strut-tie model for Beam 1. Because the model is symmetric, only half of the concrete struts are presented in the table. The resultant geometry of the strut-tie model is shown in Figure 7.

Table 2: Effective strength of concrete struts in strut-tie model for Beam 1

Strut No.	Eff. Strength (/f_c')	Strut No.	Eff. Strength (/f_c')
8,23	0.81	24,30	0.81
10,21	0.92	25,29	0.86
12,19	0.61	26,28	1.00
14,17	0.16	27	0.92
15,16	0.35	31,32	0.90

f_c': uniaxial strength of concrete

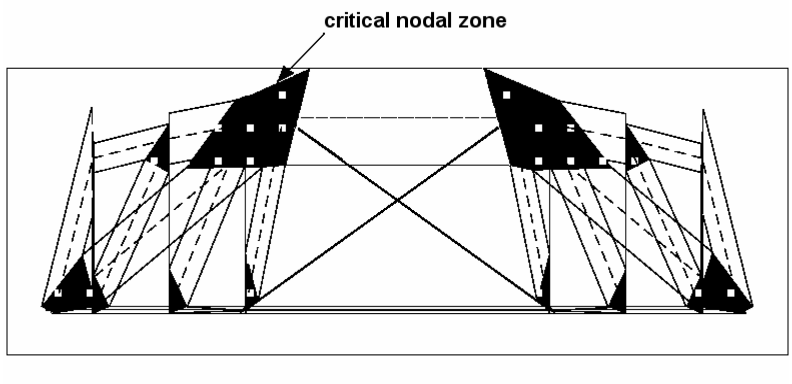


Figure 7. Dimensioned strut-tie model for Beam 1

3) Nonlinear Analysis of Strut-Tie Model

The finite element linear analysis results of the strut-tie model, including given areas and Young's modulus of elasticity of the struts and ties for the failure load, showed high tensile stresses in the stirrups (ties 11,13,18,20) indicating that they yielded before failure. To represent the complete nonlinear behavior and redistribution of the stresses in the beam, the external loads were applied in small increments. In the finite element nonlinear analysis of the strut-tie model itself, both mechanisms (arch action and single diagonal strut) were active in carrying the applied load prior to the yielding of the stirrups. The yielding of stirrups 11 and 20 and stirrups 13 and 18 was predicted at a shear force of 334.9kN and 454.7kN, respectively. With a load of 406.9kN, the direct diagonal struts (31 and 32) reached their peak stresses, and the stiffness of the struts was considered to be very small such that those struts could not carry any additionally applied load at the subsequent loading steps. In contrast, the arch members with yielded stirrups continued to carry the load up to failure. The members 15 and 16 switched from concrete struts to concrete ties at a load of 95.6kN and reached their peak stresses at a load of 239.3kN. The model member forces according to the conventional and nonlinear strut-tie model approaches are listed in Table 3. The strain history of the longitudinal steel reinforcement is shown in Figure. 8. The strain behavior predicted by the nonlinear strut-tie model approach compared better with the test results than those forecast by the conventional strut-tie model approach, plus the strain behavior can be predicted even more accurately when the strut-tie model is more refined with additional concrete ties. The numerically evaluated steel strains were not linearly proportional to the applied load because the direct diagonal struts and concrete ties, after reaching their peak compressive and tensile strains, respectively, do not carry any subsequent incremental loads inducing load redistributions of the strut-tie model.

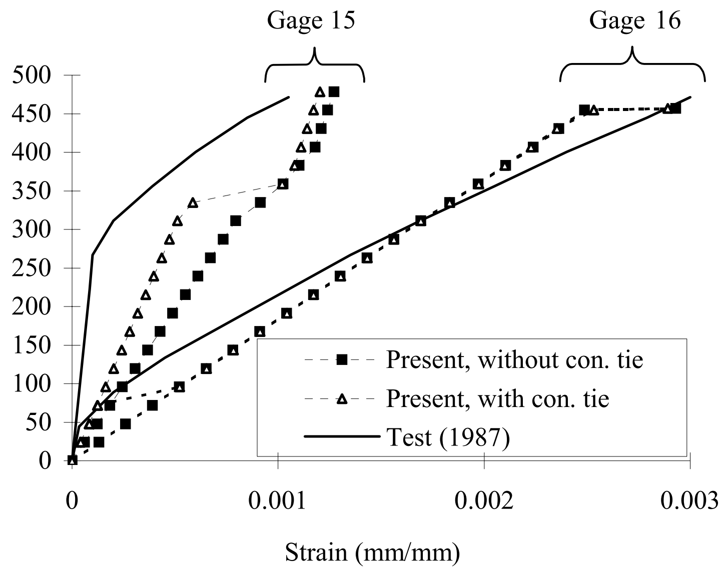


Figure 8.